Uncertainty Quantification

Karlsruhe Institute of Technology

Master Thesis:
SGD meets MDE

Course of Study: Mathematics, Computational Statistics

Topic

Minimum distance estimation (MDE) is an ubiquitous method for the statistical inference in parametric problems which, in certain contexts, encapsulates many commonly known inference methods, e.g. maximum likelihood estimation, least squares, method of moments, as special cases. A very crude description of this general inference methodology is the search for a parameter that minimizes a predefined distance measure between an empirical quantity depending on given observations, i.e. the data, and a theoretical quantity that defines the distributional model of the observations. In this project we consider the following minimum distance estimation method.

Let φ be some measurable Lebesgue-density on $\mathbb{R}^d$. Define the distance

$$\Delta_T(\vartheta, X) := \int_{\mathbb{R}^d} \left| \frac{1}{T} \int_0^T e^{iu^\top X(t)}\, dt - \int_{\mathbb{R}^d} e^{iu^\top x} \mu(\vartheta, x)\, dx \right|^2 \varphi(u)\, du, \quad \vartheta \in \Theta. \tag{1}$$

Here is $\mu(\vartheta, \cdot)$ the invariant density of the Markov process X which is the unique (weak) solution to the stochastic differential equation

$$dX(t) = b(\vartheta, X(t))dt + \sigma(X(t))dW(t), \quad t \in [0, T], \quad X(0) = x_0 \in \mathbb{R}^d. \tag{2}$$

The minimum distance estimator is then defined by means of the minimization task

$$\hat{\vartheta}_T(X) := \arg\inf_{\vartheta \in \Theta} \Delta_T(\vartheta, X). \tag{3}$$

Tasks

In general, minimum distance estimators already enjoy several favourable theoretical properties such as consistency or asymptotic normality. However, the question of practicability is, in most cases, unanswered as the minimization task (3) can be computationally extremely challenging. Hence, in this master thesis project there will be a particular focus on making the MDE computable by clever choice of the weight φ and suitable transformation of the distance (1) into a setting amenable to stochastic gradient descent (SGD) / stochastic approximation as it was originally introduced by Robbins-Monro. The developed algorithms may be implemented in any preferred language of choice. A comprehensive simulation study will constitute the biggest fraction of this project, but further theoretical analysis is possible, as well, although not necessary.

What we offer

A master thesis project close to recent research developments with the opportunity to bring in own ideas and a potential prospect of publication in a scientific journal.

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