Karlsruher Institut für Technologie

Master's Thesis

Discontinuous Galerkin Methods with Entropy Reconstruction

Courses: Mathematics, Computer Science, Physics

Topic

Discontinuous Galerkin (DG) methods have become a main method to construct higher order discretizations for hyperbolic conservation laws. They rely on polynomials on individual grid cells, for which no global continuity or smoothness is required, and which are only coupled through fluxes over cell boundaries. This has the side-effect that DG is suitable for modern computer architectures (little communication, many local operations).
At discontinuities in the solution, DG however suffers from oscillations and can violate the maximum principle of the analytical solutions, i.e. the DG solution has over- and undershoots. A possibility to mitigate this effect are minimum entropy reconstructions, which are a well-known discretization method for the velocity variable in a kinetic equation. These are used to reconstruct an ansatz function in each spatial cell, given the moments and fluxes. The reconstruction is obtained by solving a constrained optimization problem in each spatial cell. The method has the advantage that key properties of the solution can be encoded in the entropy (the quantity to be minimized), and can thus be guaranteed for the numerical solution.

Task

The task of this thesis project is to derive, study, and implement a DG scheme for scalar hyperbolic conservation laws. The individual steps are:

- Derive a suitable DG formulation with entropy reconstruction
- Investigate properties of the numerical scheme
- Implement the scheme and perform experiments with different entropies

The thesis can focus on a subset of these questions.

Contact

Prof. Dr. Martin Frank, Jonas Kusch
Faculty of Mathematics
Steinbuch Centre for Computing
www.scc.kit.edu
martin.frank@kit.edu, jonas.kusch@kit.edu